Question 3.

Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having same magnitude such that the angle between them is 60° and their scalar product is 9/2.

$\vec{a} \cdot \vec{b} = \frac{9}{2}$

$|\vec{a}| = |\vec{b}|$ $\quad \theta = 60^\circ$

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$\frac{9}{2} = |\vec{a}|\,|\vec{a}| \cos 60^\circ$

$\frac{9}{2} = |\vec{a}|^2 \times \frac{1}{2}$

$\frac{9}{2} \times \frac{2}{1} = |\vec{a}|^2$

$9 = |\vec{a}|^2$

$\sqrt{9} = |\vec{a}|$

$|\vec{a}| = 3$

$|\vec{b}| = 3$

Question 4.

(i) If $|\vec{a}| = 2$, $|\vec{b}| = \sqrt{3}$ and $\vec{a} \cdot \vec{b} = 3$, then find the angle between $\vec{a}$ and $\vec{b}$.

(ii) Find the angle between the vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 1$.

(iii) Find the angle between two vectors having the same length $\sqrt{2}$ and scalar product -1.

(iv) If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $60°$, find $\vec{a} \cdot \vec{b}$

(i)

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$\boxed{\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}} \checkmark$$

$|\vec{a}| = 2 \quad |\vec{b}| = \sqrt{3} \quad \vec{a} \cdot \vec{b} = 3$

$$\cos\theta = \frac{3}{2 \times \sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\cos\theta = \frac{3\sqrt{3}}{2 \times 3}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \cos 30°$$

$$\boxed{\theta = 30°}$$

$|\vec{a}| = |\vec{b}| = 3 \qquad \vec{a} \cdot \vec{b} = 1$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\theta = \frac{1}{3 \times 3}$$

$$\cos\theta = \frac{1}{9}$$

$$\boxed{\theta = \cos^{-1}\frac{1}{9}}$$

$|\vec{a}| = |\vec{b}| = \sqrt{2}$ $\quad \vec{a} \cdot \vec{b} = -1$

$$\cos\theta = \frac{-1}{\sqrt{2} \times \sqrt{2}}$$

$$\cos\theta = -\frac{1}{2}$$

$$\boxed{\theta = -60^\circ}$$

$|\vec{a}| = \sqrt{3} \quad |\vec{b}| = 2 \quad \vec{a}\cdot\vec{b} = ? \quad \theta = 60^\circ$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\vec{a}\cdot\vec{b} = \sqrt{3} \times 2 \times \cos 60^\circ$

$\vec{a}\cdot\vec{b} = \cancel{2}\sqrt{3} \times \frac{1}{\cancel{2}}$

$$\boxed{\vec{a}\cdot\vec{b} = \sqrt{3}}$$

Question 5.

Find the angle between the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$ (NCERT Exemplar)

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = 3\hat{i} + 4\hat{j} - \hat{k}$$

$$\vec{a} \cdot \vec{b} = (6 - 4 - 1) = (6 - 5) = 1$$

$$|\vec{a}| = \sqrt{4 + 1 + 1} = \sqrt{6}$$

$$|\vec{b}| = \sqrt{9 + 16 + 1} = \sqrt{26}$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{1}{\sqrt{6} \times \sqrt{26}}$$

$$\cos\theta = \frac{1}{\sqrt{6} \times \sqrt{26}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{6}\sqrt{26}}\right)$$

Question 6.

Find

(i) $(\vec{b}-\vec{a})\cdot(3\vec{a}+\vec{b})$ where $\vec{a} = \hat{i} - 2\hat{j} + 5\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$

(ii) $(\vec{a}+3\vec{b})\cdot(2\vec{a}-\vec{b})$ where $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$

$$(\vec{b}-\vec{a}) = 1\hat{i} + 3\hat{j} - 8\hat{k}$$

$$3\vec{a}+\vec{b} = 3\hat{i} - 6\hat{j} + 15\hat{k} + 2\hat{i} + \hat{j} - 3\hat{k}$$

$$= 5\hat{i} - 5\hat{j} + 12\hat{k}$$

$$(\vec{b}-\vec{a})\cdot(3\vec{a}+\vec{b}) = 5 - 15 - 96 = 5 - 111 = -106 \text{ Ans}$$